High School Content Expectations

MATHEMATICS

- Quantitative Literacy and Logic
- Algebra and Functions
- Geometry and Trigonometry
- Statistics and Probability
Mathematics Work Group

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Why Develop Content Standards and Expectations for High School?
In 2004, the Michigan Department of Education embraced the challenge to initiate a “high school redesign” project. Since then, the national call to create more rigorous learning for high school students has become a major priority for state leaders across the country. The Cherry Commission Report highlighted several goals for Michigan including the development of high school content expectations that reflect both a rigorous and a relevant curricular focus. Dovetailing with this call to “curricular action” is Michigan’s legislative change in high school assessment. The Michigan Merit Exam, based on rigorous high school learning standards, is to be fully implemented by 2007.

Given these two catalysts, the Michigan Department of Education’s Office of School Improvement led the development of high school content expectations for English Language Arts and Mathematics. Content area work groups of academicians chaired by a nationally known scholar in the respective field were commissioned to conduct a scholarly review and identify content standards and expectations. These content standards and expectations have gone through an extensive field and national review and are presented to educators in this document.

An Overview
The expectations contained in this document reflect best practices and current research in the teaching and learning of mathematics. They build from the Michigan Mathematics Curriculum Framework Standards and Benchmarks (1996), the Career and Employability Skills Content Standards and Benchmarks (2001), and extend the Michigan K-8 Mathematics Grade Level Content Expectations (2004) as appropriate for grades 9-12. These standards and expectations represent a vision for a rigorous and relevant high school experience for all Michigan students over the next five to ten years. Special attention has been paid to national research and support for the skills that prepare students for successful post-secondary engagement and the workplace.

The standards and expectations are closely aligned with national standards as described in ACT’s College Readiness Standards®, American Diploma Project’s Ready or Not: Creating a High School Diploma That Counts (2004), the National Council of Teachers of Mathematics Principles and Standards for School Mathematics (2000), and the National Assessment Governing Board’s Mathematics Framework for the 2003 National Assessment of Educational Progress (NAEP). Students whose work is guided by these standards and expectations will be prepared both for college and for the workplace.

Curriculum and Assessment
This document is intended to support conversations at the school and district level that result in rigorous and relevant curriculum that incorporates these content expectations.

As stakeholders (e.g., teachers, administrators, school board members, parents, community members, students, local legislative representatives) work with these standards, they should consider the following questions:

- How are these content standards and expectations reflected in our curriculum and instruction already?
- Where do we need to strengthen our curriculum and instruction to more fully realize the intent of these standards and expectations?
- What opportunities do these standards and expectations present to develop new and strengthen existing curriculum, leading to instructional excellence?
- How do we implement these standards and expectations taking into account what we know about our students, school, and community?
- How will we assess the effectiveness with which our students and schools are meeting these standards and content expectations?
- How can we use school-based assessments (e.g., student portfolios, school-based writing assessments, teacher or classroom research, district-level assessments) to make data-driven decisions about teaching and learning?

Through conversations about questions such as these, and building upon the multitude of existing strengths in our current high schools, voices of all stakeholders will participate in the important and continuing process of shaping instructional excellence in Michigan schools and preparing students in Michigan schools for college and the workplace.
Mathematics

Mathematical understandings and skills are essential elements for meaningful participation in the global information society. US expectations in mathematics for high school students have not kept pace with expectations in high-achieving countries around the world. And, expectations about who can do mathematics in the US have led to inequitable and unacceptably low opportunities to learn for students living in poor and urban communities. In Michigan, the K-8 Mathematics Grade Level Expectations represent a major step forward in raising expectations in mathematics for all students. These high school expectations assume the ambitious foundation of the K-8 GLCEs and are intended to equip all students with a solid background for continued postsecondary study in any area, as well as with skills and knowledge essential for the workplace. It is essential to hold high expectations in mathematics for all students for completion of high school, whether they will enter the workforce or go on to postsecondary education.

The high school mathematics content expectations are organized in four strands: Quantitative Literacy and Logic, Algebra and Functions, Geometry and Trigonometry, and Statistics and Probability. The topics within each strand have been arranged to show mathematical growth and to illustrate mathematical trajectories of ideas that build on one another, when possible. The expectations in these four strands are not mapped into course arrangements in this document. Such mapping, whether to traditional course titles like Algebra I, Geometry, or Algebra II, or into courses that integrate the material, is a complex process.

Decisions about the inclusion of topics were based on the following five criteria:

- how well the topic connects to other mathematical areas
- the mathematical centrality of the topic
- the standing of the topic as a cultural accomplishment
- the relevance of the topic for secondary school students
- the importance of the topic in the workplace or for informed citizenship

There is a strong emphasis on mathematical reasoning throughout all of these strands. It is also important for high school students to become successful in applying mathematical concepts and processes to solve complex problems. Technological advances affect what is possible to learn, and what is necessary to learn, in high school mathematics, and these expectations reflect this trend.

These four strands are fundamentally interconnected and also arranged to reflect the sequencing and emphases in the mathematical ideas that are central to high school.
### Understanding the Organizational Structure

The expectations in this document are divided into four strands with multiple standards within each, as shown below. The skills and content addressed in these standards will, in practice, be woven together into a coherent, integrated Mathematics curriculum. The standards are comprehensive and are meant to be used as a guide to curriculum development.

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### Core and Recommended Expectations

The expectations in this document represent what all Michigan high school graduates should know and be able to do in mathematics. With a focused and coherent set of **required core expectations**, teachers can provide both the breadth of mathematical experiences required for students to succeed in an increasingly competitive world economy, and also provide the depth required for mastery of fundamental mathematical ideas. There should be far less of the review and revisiting of topics that is typical in the high school mathematics curriculum. With a deep understanding of these expectations, students will make connections among fundamental mathematical ideas, and will be well-situated to use their mathematical knowledge and quantitative skills across the curriculum.

At the end of each strand, a set of **recommended expectations** is listed. These extensions represent content that is desirable and valuable for all students, but attention to these items should not displace or dilute the curricular emphasis of any of the core expectations. Teachers are encouraged to incorporate the recommended expectations into their instruction when their students have a solid foundation and are ready for enrichment or advanced learning. Recommended expectations may also be included in precalculus or statistics course content expectations when they are developed.

### Addendum

The addendum represents additional mathematics recommended for all students. This section provides commentary on the importance of ensuring that all students have opportunities to learn precalculus and statistics and probability, as preparation for the workplace and/or for higher education. Specific course content expectations will be developed to address the themes and topics in the **Addendum**.
Preparing Students for Successful Post-Secondary Engagement

As educators use these standards and expectations to develop rigorous and relevant units of instruction, powerful and engaging learning activities, and challenging high school curricula, it is critical to keep in mind that content knowledge alone will not provide adequate preparation for success in entry-level university courses or entry-level positions in today’s workforce.

Successful post-secondary engagement requires that students must be able to apply knowledge in new situations; to solve problems by generating new ideas; to make connections between what they read and hear in class, the world around them, and the future; and through their work, develop leadership qualities while still in high school.

Therefore, educators must model for and develop in students the cognitive skills and habits of mind that will result in mathematical proficiency and successful post-secondary engagement.

This chart includes talking points for the professional development model.
"In an increasingly complex world, adults are challenged to apply sophisticated quantitative knowledge and reasoning in their professional and personal lives. The technological demands of the workplace, the abundance of data in the political and public policy context, and the array of information involved in making personal and family decisions of all types necessitate an unprecedented facility not only with fundamental mathematical, statistical, and computing ideas and processes, but with higher-order abilities to apply and integrate those ideas and processes in a range of areas."  

The Michigan Grade Level Content Expectations in Mathematics for grades K-8 prescribe a thorough treatment of number, including strong emphasis on computational fluency and understanding of number concepts, to be completed largely by the sixth grade. The expectations in this Quantitative Literacy and Logic strand provide a definition of secondary school quantitative literacy for all students and emphasize the importance of logic as part of mathematics and in everyday life. They assume fluency (that is, efficiency and accuracy) in calculation with the basic number operations involving rational numbers in all forms (including percentages and decimals), without calculators.

Mathematical reasoning and logic are at the heart of the study of mathematics. As students progress through elementary and middle school, they increasingly are asked to explain and justify the thinking underlying their work. In high school, students peel away the contexts and study the language and thought patterns of formal mathematical reasoning. By learning logic and by constructing arguments and proofs, students will strengthen not only their knowledge and facility with mathematics, but also their ways of thinking in other areas of study and in their daily lives.

Connections and applications of number ideas and logic to other areas of mathematics, such as algebra, geometry, and statistics, are emphasized in this strand. Number representations and properties extend from the rational numbers into the real and complex numbers, as well as to other systems that students will encounter both in the workplace and in more advanced mathematics. The expectations for calculation, algorithms and estimation reflect important uses of number in a range of real-life situations. Ideas about measurement and precision tie closely to geometry.


STANDARD L1: REASONING ABOUT NUMBERS, SYSTEMS, AND QUANTITATIVE SITUATIONS

Based on their knowledge of the properties of arithmetic, students understand and reason about numbers, number systems, and the relationships between them. They represent quantitative relationships using mathematical symbols, and interpret relationships from those representations.

L1.1 Number Systems and Number Sense

L1.1.1 Know the different properties that hold in different number systems, and recognize that the applicable properties change in the transition from the positive integers, to all integers, to the rational numbers, and to the real numbers.

L1.1.2 Explain why the multiplicative inverse of a number has the same sign as the number, while the additive inverse of a number has the opposite sign.

L1.1.3 Explain how the properties of associativity, commutativity, and distributivity, as well as identity and inverse elements, are used in arithmetic and algebraic calculations.

L1.1.4 Describe the reasons for the different effects of multiplication by, or exponentiation of, a positive number by a number less than 0, a number between 0 and 1, and a number greater than 1.

L1.1.5 Justify numerical relationships (e.g., show that the sum of even integers is even; that every integer can be written as \(3m+k\), where \(k\) is 0, 1, or 2, and \(m\) is an integer; or that the sum of the first \(n\) positive integers is \(n(n+1)/2\)).

L1.1.6 Explain the importance of the irrational numbers \(\sqrt{2}\) and \(\sqrt{3}\) in basic right triangle trigonometry; the importance of \(\pi\) because of its role in circle relationships; and the role of \(e\) in applications such as continuously compounded interest.
STRAND 1: QUANTITATIVE LITERACY AND LOGIC (CONT.)

L1.2 Representations and Relationships
L1.2.1 Use mathematical symbols (e.g., interval notation, set notation, summation notation) to represent quantitative relationships and situations.
L1.2.2 Interpret representations that reflect absolute value relationships (e.g., $|x - a| \leq b$, or $a \pm b$) in such contexts as error tolerance.
L1.2.3 Use vectors to represent quantities that have magnitude and direction; interpret direction and magnitude of a vector numerically, and calculate the sum and difference of two vectors.
L1.2.4 Organize and summarize a data set in a table, plot, chart, or spreadsheet; find patterns in a display of data; understand and critique data displays in the media.

L1.3 Counting and Probabilistic Reasoning
L1.3.1 Describe, explain, and apply various counting techniques (e.g., finding the number of different 4-letter passwords; permutations; and combinations); relate combinations to Pascal’s triangle; know when to use each technique.
L1.3.2 Define and interpret commonly used expressions of probability (e.g., chances of an event, likelihood, odds).
L1.3.3 Recognize and explain common probability misconceptions such as “hot streaks” and “being due.”

STANDARD L2: CALCULATION, ALGORITHMS, AND ESTIMATION
Students calculate fluently, estimate proficiently, and describe and use algorithms in appropriate situations (e.g., approximating solutions to equations.) They understand the basic ideas of iteration and algorithms.

L2.1 Calculation Using Real and Complex Numbers
L2.1.1 Explain the meaning and uses of weighted averages (e.g., GNP, consumer price index, grade point average).
L2.1.2 Calculate fluently with numerical expressions involving exponents; use the rules of exponents; evaluate numerical expressions involving rational and negative exponents; transition easily between roots and exponents.
L2.1.3 Explain the exponential relationship between a number and its base 10 logarithm, and use it to relate rules of logarithms to those of exponents in expressions involving numbers.
L2.1.4 Know that the complex number $i$ is one of two solutions to $x^2 = -1$.
L2.1.5 Add, subtract, and multiply complex numbers; use conjugates to simplify quotients of complex numbers.
L2.1.6 Recognize when exact answers aren’t always possible or practical; use appropriate algorithms to approximate solutions to equations (e.g., to approximate square roots).

L2.2 Sequences and Iteration
L2.2.1 Find the $n$th term in arithmetic, geometric, or other simple sequences.
L2.2.2 Compute sums of finite arithmetic and geometric sequences.
L2.2.3 Use iterative processes in such examples as computing compound interest or applying approximation procedures.

STANDARD L3: MEASUREMENT AND PRECISION
Students apply measurement units and calculations, and understand the concept of error.

L3.1 Measurement Units, Calculations, and Scales
L3.1.1 Convert units of measurement within and between systems; explain how arithmetic operations on measurements affect units, and carry units through calculations correctly.
L3.1.2 Describe and interpret logarithmic relationships in such contexts as the Richter scale, the pH scale, or decibel measurements (e.g., explain why a small change in the scale can represent a large change in intensity); solve applied problems.
L3.2 Understanding Error
L3.2.1 Determine what degree of accuracy is reasonable for measurements in a given situation; express accuracy through use of significant digits, error tolerance, or percent of error; describe how errors in measurements are magnified by computation; recognize accumulated error in applied situations.

L3.2.2 Describe and explain round-off error, rounding, and truncating.

L3.2.3 Know the meaning of and interpret statistical significance, margin of error, and confidence level.

STANDARD L4: MATHEMATICAL REASONING, LOGIC, AND PROOF
Students understand mathematical reasoning as being grounded in logic and proof and can distinguish mathematical arguments from other types of arguments. They can interpret arguments made about quantitative situations in the popular media. Students know the language and laws of logic and can apply them in both mathematical and everyday settings. They write proofs using direct and indirect methods and use counterexamples appropriately to show that statements are false.

L4.1 Mathematical Reasoning
L4.1.1 Distinguish between inductive and deductive reasoning, identifying and providing examples of each.

L4.1.2 Differentiate between statistical arguments (statements verified empirically using examples or data) and logical arguments based on the rules of logic.

L4.1.3 Define and explain the roles of axioms (postulates), definitions, theorems, counterexamples, and proofs in the logical structure of mathematics; identify and give examples of each.

L4.2 Language and Laws of Logic
L4.2.1 Know and use the terms of basic logic (e.g., proposition, negation, truth and falsity, implication, if and only if, contrapositive, and converse).

L4.2.2 Use the connectives “NOT,” “AND,” “OR,” and “IF…THEN,” in mathematical and everyday settings. Know the truth table of each connective and how to logically negate statements involving these connectives.

L4.2.3 Use the quantifiers “THERE EXISTS” and “ALL” in mathematical and everyday settings and know how to logically negate statements involving them.

L4.2.4 Write the converse, inverse, and contrapositive of an “If…, then…” statement; use the fact, in mathematical and everyday settings, that the contrapositive is logically equivalent to the original while the inverse and converse are not.

L4.3 Proof
L4.3.1 Know the basic structure for the proof of an “If…, then…” statement (assuming the hypothesis and ending with the conclusion) and know that proving the contrapositive is equivalent.

L4.3.2 Construct proofs by contradiction; use counterexamples, when appropriate, to disprove a statement.

L4.3.3 Explain the difference between a necessary and a sufficient condition within the statement of a theorem; determine the correct conclusions based on interpreting a theorem in which necessary or sufficient conditions in the theorem or hypothesis are satisfied.

RECOMMENDED:
* L1.2.5 Read and interpret representations from various technological sources, such as contour or isobar diagrams.
* L2.1.7 Understand the mathematical bases for the differences among voting procedures.
* L2.2.4 Compute sums of infinite geometric sequences.
In the middle grades, students see the progressive generalization of arithmetic to algebra. They learn symbolic manipulation skills and use them to solve equations. They study simple forms of elementary polynomial functions such as linear, quadratic, and power functions as represented by tables, graphs, symbols, and verbal descriptions.

In high school, students continue to develop their “symbol sense” by examining expressions, equations, and functions, and applying algebraic properties to solve equations. They construct a conceptual framework for analyzing any function and, using this framework, they revisit the functions they have studied before in greater depth. By the end of high school, their catalog of functions will encompass linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric functions. They will be able to reason about functions and their properties and solve multi-step problems that involve both functions and equation-solving. Students will use deductive reasoning to justify algebraic processes as they solve equations and inequalities, as well as when transforming expressions.

This rich learning experience in Algebra will provide opportunities for students to understand both its structure and its applicability to solving real-world problems. Students will view algebra as a tool for analyzing and describing mathematical relationships, and for modeling problems that come from the workplace, the sciences, technology, engineering, and mathematics.

**STANDARD A1: EXPRESSIONS, EQUATIONS, AND INEQUALITIES**

Students recognize, construct, interpret, and evaluate expressions. They fluently transform symbolic expressions into equivalent forms. They determine appropriate techniques for solving each type of equation, inequality, or system of equations, apply the techniques correctly to solve, justify the steps in the solutions, and draw conclusions from the solutions. They know and apply common formulas.

**A1.1 Construction, Interpretation, and Manipulation of Expressions (linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric)**

A1.1.1 Give a verbal description of an expression that is presented in symbolic form, write an algebraic expression from a verbal description, and evaluate expressions given values of the variables.

A1.1.2 Know the definitions and properties of exponents and roots, transition fluently between them, and apply them in algebraic expressions.

A1.1.3 Factor algebraic expressions using, for example, greatest common factor, grouping, and the special product identities (e.g., differences of squares and cubes).

A1.1.4 Add, subtract, multiply, and simplify polynomials and rational expressions (e.g., multiply \((x - 1)(1 - x^2 + 3)\); simplify \(\frac{9x - 2x^3}{x + 3}\)).

A1.1.5 Divide a polynomial by a monomial.

A1.1.6 Transform exponential and logarithmic expressions into equivalent forms using the properties of exponents and logarithms including the inverse relationship between exponents and logarithms.

**A1.2 Solutions of Equations and Inequalities (linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric)**

A1.2.1 Write equations and inequalities with one or two variables to represent mathematical or applied situations, and solve.

A1.2.2 Associate a given equation with a function whose zeros are the solutions of the equation.

A1.2.3 Solve (and justify steps in the solutions) linear and quadratic equations and inequalities, including systems of up to three linear equations with three unknowns; apply the quadratic formula appropriately.

A1.2.4 Solve absolute value equations and inequalities, (e.g. solve \(|x - 3| \leq 6\), and justify steps in the solution.

A1.2.5 Solve polynomial equations and equations involving rational expressions (e.g. solve \(-2x(x^2 + 4x + 3) = 0\); solve \(x - \frac{1}{x + 6} = 3\), and justify steps in the solution.

A1.2.6 Solve power equations (e.g., \((x + 1)^2 = 8\)) and equations including radical expressions (e.g., \(\sqrt{3x - 7} = 7\)), justify steps in the solution, and explain how extraneous solutions may arise.
A1.2.7 Solve exponential and logarithmic equations (e.g., \(3(2^x) = 24\), \(2 \ln(x + 1) = 4\)), and justify steps in the solution.

A1.2.8 Solve an equation involving several variables (with numerical or letter coefficients) for a designated variable, and justify steps in the solution.

A1.2.9 Know common formulas (e.g., slope, distance between two points, quadratic formula, compound interest, distance = velocity • time), and apply appropriately in contextual situations.

A1.2.10 Use special values of the inverse trigonometric functions to solve trigonometric equations over specific intervals (e.g., \(2 \sin x - 1 = 0\) for \(0 \leq x \leq 2\pi\)).

STANDARD A2: FUNCTIONS

Students understand functions, their representations, and their attributes. They perform transformations, combine and compose functions, and find inverses. Students classify functions and know the characteristics of each family. They work with functions with real coefficients fluently.

A2.1 Definitions, Representations, and Attributes of Functions

A2.1.1 Recognize whether a relationship (given in contextual, symbolic, tabular, or graphical form) is a function; and identify its domain and range.

A2.1.2 Read, interpret, and use function notation, and evaluate a function at a value in its domain.

A2.1.3 Represent functions in symbols, graphs, tables, diagrams, or words, and translate among representations.

A2.1.4 Recognize that functions may be defined by different expressions over different intervals of their domains; such functions are piecewise-defined (e.g., absolute value and greatest integer functions).

A2.1.5 Recognize that functions may be defined recursively, and compute values of and graph simple recursively defined functions (e.g., \(f(0) = 5\), and \(f(n) = f(n-1) + 2\)).

A2.1.6 Identify the zeros of a function and the intervals where the values of a function are positive or negative, and describe the behavior of a function, as \(x\) approaches positive or negative infinity, given the symbolic and graphical representations.

A2.1.7 Identify and interpret the key features of a function from its graph or its formula(e), (e.g. slope, intercept(s), asymptote(s), maximum and minimum value(s), symmetry, average rate of change over an interval, and periodicity).

A2.2 Operations and Transformations

A2.2.1 Combine functions by addition, subtraction, multiplication, and division.

A2.2.2 Apply given transformations (e.g., vertical or horizontal shifts, stretching or shrinking, or reflections about the \(x\)- and \(y\)-axes) to basic functions, and represent symbolically.

A2.2.3 Recognize whether a function (given in tabular or graphical form) has an inverse and recognize simple inverse pairs (e.g., \(f(x) = x^3\) and \(g(x) = x^{1/3}\)).

A2.3 Families of Functions (linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric)

A2.3.1 Identify a function as a member of a family of functions based on its symbolic, or graphical representation; recognize that different families of functions have different asymptotic behavior at infinity, and describe these behaviors.

A2.3.2 Describe the tabular pattern associated with functions having constant rate of change (linear); or variable rates of change.

A2.3.3 Write the general symbolic forms that characterize each family of functions. (e.g., \(f(t) = A_0 e^{\lambda t}\); \(f(t) = A_0 e^{\lambda t}\))
A2.4 Lines and Linear Functions

A2.4.1 Write the symbolic forms of linear functions (standard [i.e., \( Ax + By = C \), where \( B \neq 0 \)], point-slope, and slope-intercept) given appropriate information, and convert between forms.

A2.4.2 Graph lines (including those of the form \( x = h \) and \( y = k \)) given appropriate information.

A2.4.3 Relate the coefficients in a linear function to the slope and \( x \)- and \( y \)-intercepts of its graph.

A2.4.4 Find an equation of the line parallel or perpendicular to given line, through a given point; understand and use the facts that non-vertical parallel lines have equal slopes, and that non-vertical perpendicular lines have slopes that multiply to give -1.

A2.5 Exponential and Logarithmic Functions

A2.5.1 Write the symbolic form and sketch the graph of an exponential function given appropriate information. (e.g., given an initial value of 4 and a rate of growth of 1.5, write \( f(x) = 4 \cdot (1.5)^x \)).

A2.5.2 Interpret the symbolic forms and recognize the graphs of exponential and logarithmic functions (e.g., \( f(x) = 10^x \), \( f(x) = \log x \), \( f(x) = e^x \), \( f(x) = \ln x \)); recognize the logarithmic function as the inverse of the exponential function.

A2.5.3 Apply properties of exponential and logarithmic functions (e.g., \( a^{x+y} = a^x a^y \); \( \log(ab) = \log a + \log b \)).

A2.5.4 Understand and use the fact that the base of an exponential function determines whether the function increases or decreases and understand how the base affects the rate of growth or decay.

A2.5.5 Relate exponential and logarithmic functions to real phenomena, including half-life and doubling time.

A2.6 Quadratic Functions

A2.6.1 Write the symbolic form and sketch the graph of a quadratic function given appropriate information (e.g., vertex, intercepts, etc.).

A2.6.2 Identify the elements of a parabola (vertex, axis of symmetry, direction of opening) given its symbolic form or its graph, and relate these elements to the coefficient(s) of the symbolic form of the function.

A2.6.3 Convert quadratic functions from standard to vertex form by completing the square.

A2.6.4 Relate the number of real solutions of a quadratic equation to the graph of the associated quadratic function.

A2.6.5 Express quadratic functions in vertex form to identify their maxima or minima, and in factored form to identify their zeros.

A2.7 Power Functions (including roots, cubics, quartics, etc.)

A2.7.1 Write the symbolic form and sketch the graph of power functions.

A2.7.2 Express direct and inverse relationships as functions (e.g., \( y = kx^n \) and \( y = kx^{-n} \), \( n > 0 \)) and recognize their characteristics (e.g., in \( y = x^3 \), note that doubling \( x \) results in multiplying \( y \) by a factor of 8).

A2.7.3 Analyze the graphs of power functions, noting reflectional or rotational symmetry.

A2.8 Polynomial Functions

A2.8.1 Write the symbolic form and sketch the graph of simple polynomial functions.

A2.8.2 Understand the effects of degree, leading coefficient, and number of real zeros on the graphs of polynomial functions of degree greater than 2.

A2.8.3 Determine the maximum possible number of zeros of a polynomial function, and understand the relationship between the \( x \)-intercepts of the graph and the factored form of the function.
A2.9 Rational Functions

A2.9.1 Write the symbolic form and sketch the graph of simple rational functions.

A2.9.2 Analyze graphs of simple rational functions (e.g., \( f(x) = \frac{2x + 1}{x - 1} \); \( g(x) = \frac{x}{x^2 - 4} \)) and understand the relationship between the zeros of the numerator and denominator and the function’s intercepts, asymptotes, and domain.

A2.10 Trigonometric Functions

A2.10.1 Use the unit circle to define sine and cosine; approximate values of sine and cosine (e.g., \( \sin 3 \) or \( \cos 0.5 \)); use sine and cosine to define the remaining trigonometric functions; explain why the trigonometric functions are periodic.

A2.10.2 Use the relationship between degree and radian measures to solve problems.

A2.10.3 Use the unit circle to determine the exact values of sine and cosine, for integer multiples of \( \pi/6 \) and \( \pi/4 \).

A2.10.4 Graph the sine and cosine functions; analyze graphs by noting domain, range, period, amplitude, and location of maxima and minima.

A2.10.5 Graph transformations of basic trigonometric functions (involving changes in period, amplitude, and midline) and understand the relationship between constants in the formula and the transformed graph.

STANDARD A3: MATHEMATICAL MODELING

Students construct or select a function to model a real-world situation in order to solve applied problems. They draw on their knowledge of families of functions to do so.

A3.1 Models of Real-world Situations Using Families of Functions.

Example: An initial population of 300 people grows at 2% per year. What will the population be in 10 years?

A3.1.1 Identify the family of function best suited for modeling a given real-world situation (e.g., quadratic functions for motion of an object under the force of gravity; exponential functions for compound interest; trigonometric functions for periodic phenomena. In the example above, recognize that the appropriate general function is exponential \((P = P_0e^{rt})\)."

A3.1.2 Adapt the general symbolic form of a function to one that fits the specifications of a given situation by using the information to replace arbitrary constants with numbers. In the example above, substitute the given values \(P_0 = 300\) and \(r = 0.02\) to obtain \(P = 300(1.02)^t\).

A3.1.3 Using the adapted general symbolic form, draw reasonable conclusions about the situation being modeled. In the example above, the exact solution is 365.698, but for this problem an appropriate approximation is 365.

RECOMMENDED:

*A1.1.7 Transform trigonometric expressions into equivalent forms using basic identities such as \(\sin^2\theta + \cos^2\theta = 1\), \(\tan\theta = \frac{\sin\theta}{\cos\theta}\) and \(\tan^2\theta + 1 = \sec^2\theta\).

*A2.2.4 If a function has an inverse, find the expression(s) for the inverse.

*A2.2.5 Write an expression for the composition of one function with another; recognize component functions when a function is a composition of other functions.

*A2.2.6 Know and interpret the function notation for inverses and verify that two functions are inverses using composition.

*A3.1.4 Use methods of linear programming to represent and solve simple real-life problems.
STRAND 3: GEOMETRY AND TRIGONOMETRY (G)

In Grades K–5, students study figures such as triangles, rectangles, circles, rectangular solids, cylinders, and spheres. They examine similarities and differences between geometric shapes. They learn to quantify geometric figures by measuring and calculating lengths, angles, areas and volumes. In Grades 6-8, students broaden their understanding of area and volume and develop the basic concepts of congruence, similarity, symmetry and the Pythagorean Theorem. They apply these ideas to solve geometric problems, including ones related to the real world.

In Grades 9–12, students see geometry developed as a coherent, structured subject. They use the geometric skills and ideas introduced earlier, such as congruence and similarity, to solve a wide variety of problems. There is an emphasis on the importance of clear language (e.g. for postulates, definitions and theorems) and on learning to construct geometric proofs. In this process, students build geometric intuition and facility at deductive reasoning. They use elements of logic and reasoning as described in the Quantitative Literacy and Logic strand, including both direct and indirect proof presented in narrative form. They begin to use new techniques, including transformations and trigonometry. They apply these ideas to solve complex problems about two- and three-dimensional figures, again including ones related to the real world. Their spatial visualization skills will be developed through the study of the relationships between two- and three-dimensional shapes.

STANDARD G1: FIGURES AND THEIR PROPERTIES

Students represent basic geometric figures, polygons, and conic sections and apply their definitions and properties in solving problems and justifying arguments, including constructions and representations in the coordinate plane. Students represent three-dimensional figures, understand the concepts of volume and surface area, and use them to solve problems. They know and apply properties of common three-dimensional figures.

G1.1 Lines and Angles; Basic Euclidean and Coordinate Geometry

G1.1.1 Solve multi-step problems and construct proofs involving vertical angles, linear pairs of angles, supplementary angles, complementary angles, and right angles.

G1.1.2 Solve multi-step problems and construct proofs involving corresponding angles, alternate interior angles, alternate exterior angles, and same-side (consecutive) interior angles.

G1.1.3 Perform and justify constructions, including midpoint of a line segment and bisector of an angle, using straightedge and compass.

G1.1.4 Given a line and a point, construct a line through the point that is parallel to the original line using straightedge and compass; given a line and a point, construct a line through the point that is perpendicular to the original line; justify the steps of the constructions.

G1.1.5 Given a line segment in terms of its endpoints in the coordinate plane, determine its length and midpoint.

G1.1.6 Recognize Euclidean Geometry as an axiom system; know the key axioms and understand the meaning of and distinguish between undefined terms (e.g., point, line, plane), axioms, definitions, and theorems.

G1.2 Triangles and Their Properties

G1.2.1 Prove that the angle sum of a triangle is 180° and that an exterior angle of a triangle is the sum of the two remote interior angles.

G1.2.2 Construct and justify arguments and solve multi-step problems involving angle measure, side length, perimeter, and area of all types of triangles.

G1.2.3 Know a proof of the Pythagorean Theorem and use the Pythagorean Theorem and its converse to solve multi-step problems.
G1.2.4 Prove and use the relationships among the side lengths and the angles of 30°- 60°- 90° triangles and 45°- 45°- 90° triangles.

G1.2.5 Solve multi-step problems and construct proofs about the properties of medians, altitudes, and perpendicular bisectors to the sides of a triangle, and the angle bisectors of a triangle; using a straightedge and compass, construct these lines.

G1.3 Triangles and Trigonometry

G1.3.1 Define the sine, cosine, and tangent of acute angles in a right triangle as ratios of sides; solve problems about angles, side lengths, or areas using trigonometric ratios in right triangles.

G1.3.2 Know and use the Law of Sines and the Law of Cosines and use them to solve problems; find the area of a triangle with sides $a$ and $b$ and included angle $\theta$ using the formula Area = $(1/2) ab \sin \theta$.

G1.3.3 Determine the exact values of sine, cosine, and tangent for $0^\circ$, $30^\circ$, $45^\circ$, $60^\circ$, and their integer multiples, and apply in various contexts.

G1.4 Quadrilaterals and Their Properties

G1.4.1 Solve multi-step problems and construct proofs involving angle measure, side length, diagonal length, perimeter, and area of squares, rectangles, parallelograms, kites, and trapezoids.

G1.4.2 Solve multi-step problems and construct proofs involving quadrilaterals (e.g., prove that the diagonals of a rhombus are perpendicular) using Euclidean methods or coordinate geometry.

G1.4.3 Describe and justify hierarchical relationships among quadrilaterals, (e.g. every rectangle is a parallelogram).

G1.4.4 Prove theorems about the interior and exterior angle sums of a quadrilateral.

G1.5 Other Polygons and Their Properties

G1.5.1 Know and use subdivision or circumscription methods to find areas of polygons (e.g., regular octagon, non-regular pentagon).

G1.5.2 Know, justify, and use formulas for the perimeter and area of a regular $n$-gon and formulas to find interior and exterior angles of a regular $n$-gon and their sums.

G1.6 Circles and Their Properties

G1.6.1 Solve multi-step problems involving circumference and area of circles.

G1.6.2 Solve problems and justify arguments about chords (e.g., if a line through the center of a circle is perpendicular to a chord, it bisects the chord) and lines tangent to circles (e.g., a line tangent to a circle is perpendicular to the radius drawn to the point of tangency).

G1.6.3 Solve problems and justify arguments about central angles, inscribed angles and triangles in circles.

G1.6.4 Know and use properties of arcs and sectors, and find lengths of arcs and areas of sectors.

G1.7 Conic Sections and Their Properties

G1.7.1 Find an equation of a circle given its center and radius; given the equation of a circle, find its center and radius.

G1.7.2 Identify and distinguish among geometric representations of parabolas, circles, ellipses, and hyperbolas; describe their symmetries, and explain how they are related to cones.

G1.7.3 Graph ellipses and hyperbolas with axes parallel to the x- and y-axes, given equations.

G1.8 Three-Dimensional Figures

G1.8.1 Solve multi-step problems involving surface area and volume of pyramids, prisms, cones, cylinders, hemispheres, and spheres.

G1.8.2 Identify symmetries of pyramids, prisms, cones, cylinders, hemispheres, and spheres.
STANDARD G2: RELATIONSHIPS BETWEEN FIGURES

Students use and justify relationships between lines, angles, area and volume formulas, and 2- and 3-dimensional representations. They solve problems and provide proofs about congruence and similarity.

G2.1 Relationships Between Area and Volume Formulas

G2.1.1 Know and demonstrate the relationships between the area formula of a triangle, the area formula of a parallelogram, and the area formula of a trapezoid.

G2.1.2 Know and demonstrate the relationships between the area formulas of various quadrilaterals (e.g., explain how to find the area of a trapezoid based on the areas of parallelograms and triangles).

G2.1.3 Know and use the relationship between the volumes of pyramids and prisms (of equal base and height) and cones and cylinders (of equal base and height).

G2.2 Relationships Between Two-dimensional and Three-dimensional Representations

G2.2.1 Identify or sketch a possible 3-dimensional figure, given 2-dimensional views (e.g., nets, multiple views); create a 2-dimensional representation of a 3-dimensional figure.

G2.2.2 Identify or sketch cross-sections of 3-dimensional figures; identify or sketch solids formed by revolving 2-dimensional figures around lines.

G2.3 Congruence and Similarity

G2.3.1 Prove that triangles are congruent using the SSS, SAS, ASA, and AAS criteria, and for right triangles, the hypotenuse-leg criterion.

G2.3.2 Use theorems about congruent triangles to prove additional theorems and solve problems, with and without use of coordinates.

G2.3.3 Prove that triangles are similar by using SSS, SAS, and AA conditions for similarity.

G2.3.4 Use theorems about similar triangles to solve problems with and without use of coordinates.

G2.3.5 Know and apply the theorem stating that the effect of a scale factor of $k$ relating one two-dimensional figure to another or one three-dimensional figure to another, on the length, area, and volume of the figures is to multiply each by $k$, $k^2$, and $k^3$, respectively.
STANDARD G3: TRANSFORMATIONS OF FIGURES IN THE PLANE

Students will solve problems about distance-preserving transformations and shape-preserving transformations. The transformations will be described synthetically and, in simple cases, by analytic expressions in coordinates.

G3.1 Distance-preserving Transformations: Isometries

G3.1.1 Define reflection, rotation, translation, and glide reflection and find the image of a figure under a given isometry.

G3.1.2 Given two figures that are images of each other under an isometry, find the isometry and describe it completely.

G3.1.3 Find the image of a figure under the composition of two or more isometries, and determine whether the resulting figure is a reflection, rotation, translation, or glide reflection image of the original figure.

G3.2 Shape-preserving Transformations: Dilations and Isometries

G3.2.1 Know the definition of dilation, and find the image of a figure under a given dilation.

G3.2.2 Given two figures that are images of each other under some dilation, identify the center and magnitude of the dilation.

RECOMMENDED:

*G1.4.5 Understand the definition of a cyclic quadrilateral, and know and use the basic properties of cyclic quadrilaterals.

*G1.7.4 Know and use the relationship between the vertices and foci in an ellipse, the vertices and foci in a hyperbola, and the directrix and focus in a parabola; interpret these relationships in applied contexts.

*G3.2.3 Find the image of a figure under the composition of a dilation and an isometry.
In Kindergarten through Grade 8, students develop the ability to read, analyze, and construct a repertoire of statistical graphs. Students also examine the fundamentals of experimental and theoretical probability in informal ways. The Basic Counting Principle and tree diagrams serve as tools to solve simple counting problems in these grades.

During high school, students build on that foundation. They develop the data interpretation and decision-making skills that will serve them in their further study of mathematics as well as in their coursework in the physical, biological, and social sciences. Students learn important skills related to the collection, display, and interpretation of both univariate and bivariate data. They understand basic sampling methods and apply principles of effective data analysis and data presentation. These skills are also highly valuable outside of school, both in the workplace and in day-to-day life.

In probability, students utilize probability models to calculate probabilities and make decisions. The normal distribution and its properties are studied. Students then use their understanding of probability to make decisions, solve problems, and determine whether or not statements about probabilities of events are reasonable. Students use technology when appropriate, including spreadsheets. This strong background in statistics and probability will enable students to be savvy decision-makers and smart information-consumers and producers who have a full range of tools in order to make wise choices.

**STANDARD S1: UNIVARIATE DATA – EXAMINING DISTRIBUTIONS**

Students plot and analyze univariate data by considering the shape of distributions and analyzing outliers; they find and interpret commonly-used measures of center and variation; and they explain and use properties of the normal distribution.

**S1.1 Producing and Interpreting Plots**

- **S1.1.1** Construct and interpret dot plots, histograms, relative frequency histograms, bar graphs, basic control charts, and box plots with appropriate labels and scales; determine which kinds of plots are appropriate for different types of data; compare data sets and interpret differences based on graphs and summary statistics.

- **S1.1.2** Given a distribution of a variable in a data set, describe its shape, including symmetry or skewness, and state how the shape is related to measures of center (mean and median) and measures of variation (range and standard deviation) with particular attention to the effects of outliers on these measures.

**S1.2 Measures of Center and Variation**

- **S1.2.1** Calculate and interpret measures of center including: mean, median, and mode; explain uses, advantages and disadvantages of each measure given a particular set of data and its context.

- **S1.2.2** Estimate the position of the mean, median, and mode in both symmetrical and skewed distributions, and from a frequency distribution or histogram.

- **S1.2.3** Compute and interpret measures of variation, including percentiles, quartiles, interquartile range, variance, and standard deviation.

**S1.3 The Normal Distribution**

- **S1.3.1** Explain the concept of distribution and the relationship between summary statistics for a data set and parameters of a distribution.

- **S1.3.2** Describe characteristics of the normal distribution, including its shape and the relationships among its mean, median, and mode.

- **S1.3.3** Know and use the fact that about 68%, 95%, and 99.7% of the data lie within one, two, and three standard deviations of the mean, respectively in a normal distribution.

- **S1.3.4** Calculate z-scores, use z-scores to recognize outliers, and use z-scores to make informed decisions.
STANDARD S2: BIVARIATE DATA – EXAMINING RELATIONSHIPS

Students plot and interpret bivariate data by constructing scatterplots, recognizing linear and nonlinear patterns, and interpreting correlation coefficients; they fit and interpret regression models, using technology as appropriate.

S2.1 Scatterplots and Correlation
   S2.1.1 Construct a scatterplot for a bivariate data set with appropriate labels and scales.
   S2.1.2 Given a scatterplot, identify patterns, clusters, and outliers; recognize no correlation, weak correlation, and strong correlation.
   S2.1.3 Estimate and interpret Pearson’s correlation coefficient for a scatterplot of a bivariate data set; recognize that correlation measures the strength of linear association.
   S2.1.4 Differentiate between correlation and causation; know that a strong correlation does not imply a cause-and-effect relationship; recognize the role of lurking variables in correlation.

S2.2 Linear Regression
   S2.2.1 For bivariate data which appear to form a linear pattern, find the least squares regression line by estimating visually and by calculating the equation of the regression line; interpret the slope of the equation for a regression line.
   S2.2.2 Use the equation of the least squares regression line to make appropriate predictions.

STANDARD S3: SAMPLES, SURVEYS, AND EXPERIMENTS

Students understand and apply sampling and various sampling methods, examine surveys and experiments, identify bias in methods of conducting surveys, and learn strategies to minimize bias. They understand basic principles of good experimental design.

S3.1 Data Collection and Analysis
   S3.1.1 Know the meanings of a sample from a population and a census of a population, and distinguish between sample statistics and population parameters.
   S3.1.2 Identify possible sources of bias in data collection and sampling methods and simple experiments; describe how such bias can be reduced and controlled by random sampling; explain the impact of such bias on conclusions made from analysis of the data; and know the effect of replication on the precision of estimates.
   S3.1.3 Distinguish between an observational study and an experimental study, and identify, in context, the conclusions that can be drawn from each.

STANDARD S4: PROBABILITY MODELS AND PROBABILITY CALCULATION

Students understand probability and find probabilities in various situations, including those involving compound events, using diagrams, tables, geometric models and counting strategies; they apply the concepts of probability to make decisions.

S4.1 Probability
   S4.1.1 Understand and construct sample spaces in simple situations (e.g., tossing two coins, rolling two number cubes and summing the results).
   S4.1.2 Define mutually exclusive events, independent events, dependent events, compound events, complementary events and conditional probabilities; and use the definitions to compute probabilities.

S4.2 Application and Representation
   S4.2.1 Compute probabilities of events using tree diagrams, formulas for combinations and permutations, Venn diagrams, or other counting techniques.
   S4.2.2 Apply probability concepts to practical situations, in such settings as finance, health, ecology, or epidemiology, to make informed decisions.
**RECOMMENDED:**

*S3.1.4*  Design simple experiments or investigations to collect data to answer questions of interest; interpret and present results.

*S3.1.5*  Understand methods of sampling, including random sampling, stratified sampling, and convenience samples, and be able to determine, in context, the advantages and disadvantages of each.

*S3.1.6*  Explain the importance of randomization, double-blind protocols, replication, and the placebo effect in designing experiments and interpreting the results of studies.

*S3.2.1*  Explain the basic ideas of statistical process control, including recording data from a process over time.

*S3.2.2*  Read and interpret basic control charts; detect patterns and departures from patterns.

*S4.1.3*  Design and carry out an appropriate simulation using random digits to estimate answers to questions about probability; estimate probabilities using results of a simulation; compare results of simulations to theoretical probabilities.
Additional Mathematics Recommended for All Students

In addition to the core and recommended expectations, we provide here a set of fundamental topics in the areas of precalculus and statistics and probability. Study in these areas is recommended as preparation for post-secondary education opportunities, as well as for the workplace.

Precalculus

Precalculus is the preparation for calculus. Calculus is the most powerful, useful and versatile branch of mathematics. While the core ideas of calculus (derivatives and integrals) are not hard to understand, calculus is a demanding subject because it requires a broad and thorough background of algebra and functions. This material is essential for college-bound students. It is a prerequisite for many college programs in science, engineering, medicine, and business.

Students study precalculus in order to deeply understand, make connections among, and apply the topics introduced in algebra. Those algebraic topics are now treated at a higher conceptual level. The theory and applications of trigonometry and functions are developed in depth. New mathematical tools, such as vectors, matrices and polar coordinates, are introduced with an eye toward modeling and solving real world problems.

• Functions

Students use the definition of function (including domain and range). They combine functions using algebraic operations and composition. They write a given function as a composition of simpler functions. The notion of one-to-one function is introduced, leading to the definition of an inverse function. Students find the symbolic expression for the inverse of a given function and show that two given functions are inverses.

• Exponential and logarithmic functions

Students graph logarithmic functions as inverses of exponential functions, and solve equations involving exponential and logarithmic functions. They determine the asymptotic behavior of exponential and logarithmic functions with different bases. Students apply these functions in real world situations, such as exponential growth and decay, and compound interest.

• Trigonometric functions and analytical trigonometry

Students use the unit circle to define sine, cosine, and the other trigonometric functions. They apply transformations involving changes in amplitude, midline, period, and phase, to trigonometric functions and represent the results graphically and symbolically. The inverse trigonometric functions and their graphs are introduced. They establish and learn sum and difference formulas and other basic trigonometric identities. They use these to simplify trigonometric expressions, to solve trigonometric equations, prove trigonometric identities, and solve applied problems.

• Polynomial and rational functions

Students learn the Fundamental Theorem of Algebra, the Remainder Theorem, and the Factor Theorem. They solve polynomial equations and inequalities by factoring and dividing polynomials. They identify the large-scale behavior of the graph of a polynomial from its leading term. Students solve rational equations and inequalities. They determine the asymptotes of the graph of a rational function.
• **Difference quotients and limits**
  Students learn the definition and geometric interpretation of difference quotients. Using the definition, they represent and simplify difference quotients and interpret difference quotients as rates of change and slopes of secant lines. They acquire an informal meaning for the limit of a function and relate that meaning to the graph.

• **Vectors and matrices**
  Students sketch and perform operations (multiplication by scalars, addition, and subtraction) of vectors in the plane and use vectors in applications. They learn the algebraic and geometric definitions of dot product of vectors, and use them in applications. Students represent rotations of the plane as matrices and apply these in the context of analytic geometry. They multiply matrices and multiply vectors by matrices, compute determinants, and they solve systems of two and three linear equations by matrix methods. Students compute inverses of three-by-three matrices, when they exist.

• **Sequences and series**
  Students find the nth term in arithmetic sequences, geometric sequences, and recursively defined sequences. They use sigma notation and compute sums of finite arithmetic sequences. Students compute sums of finite and infinite geometric series and apply the convergence criterion for geometric series.

• **Polar coordinates, parameterizations, and conic sections**
  Students use polar coordinates and graph equations in polar form. They write complex numbers in polar form and use DeMoivre’s Theorem. Students parameterize segments and curves, and recognize implicitly defined curves. They identify parabolas, ellipses and hyperbolas from their equations, put the equations in standard form, sketch and analyze the graphs and characteristics (e.g. finding foci).

• **Mathematical reasoning**
  Students prove theorems and use mathematical induction.

**Statistics and Probability**

Students encounter variability in their lives, in their science and social studies coursework, and in the news media. The study of statistics and probability gives students methods for summarizing data, introduces students to mathematical models for random phenomena, and provides the tools for decision making under uncertainty.

Three important considerations should be kept in mind when reading the topics below. First, as much as possible, the concepts and techniques should be introduced and used in the context of specific studies. Sometimes these are called “real-world” applications, although it is often useful to clean up data before presenting it to students, so that the essential concepts are not obscured by the complexities of the data. Second, wherever possible, technology (calculators or statistical software) should be used for computations and graphing. Third, simulation can give students deeper understanding of many probability and inferential concepts, and should be used often.

The topics are built around four themes: data exploration and study design; probability models and their application; statistical inference; and model assessment. Some of the topics have been introduced in the required curriculum, and are extended and treated in more depth in the extended curriculum. More details on the specific topics are given next.
• **Exploring univariate and bivariate data**
  Students learn and apply techniques for exploring univariate and bivariate data using both graphical and numerical summaries. Fundamental is fostering students’ understanding of variability in data, and learning how to make comparisons between data sets in the presence of variability.

• **Sampling and study design**
  Students learn methods of designing surveys and controlled experiments, design surveys and experiments, and use their knowledge of design to critically assess conclusions. The importance of randomization in minimizing bias, and of methods such as blocking to reduce variability, are stressed.

• **Probability models**
  Students are introduced to commonly-used discrete probability models such as the binomial and hypergeometric models and use these to model real-world phenomena. Conditional probabilities including Bayes’ Theorem are used to solve problems from health, public policy, and other areas, and students gain facility with the normal distribution.

• **Sampling distributions**
  Students learn results for sums of random variables, including an informal treatment of the central limit theorem, and apply these results to sampling distributions of common estimators. Statistical process control and the creation and interpretation of control charts are included.

• **Point and interval estimation**
  Students study basic properties of point estimators, and then apply their knowledge of sampling distributions to construct confidence interval estimators for means and proportions in one- and two- sample problems for both means and proportions. Correct interpretation of confidence intervals is stressed.

• **Significance testing**
  Students learn the terminology and logic of significance testing, including power, learn to perform significance tests for means and proportions in one- and two-sample problems for means and proportions, and are introduced to chi-squared testing. A main focus is on correct interpretation of results.

• **Inference for regression**
  The statistical model for simple linear regression is introduced, and students learn to construct confidence intervals and perform significance tests for the slope of a regression line.

• **Assessing assumptions of statistical models**
  Throughout the sections on probability and statistical inference, the role of the underlying mathematical model and its assumptions is kept in the forefront. Students learn to assess the validity of the model and to gauge the effect of departures from model assumptions.